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BOOTSTRAP TECHNOLOGY FOR RAM ANALYSIS

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1. Introduction

The Bootstrap technology is one of the most powerful tools for modern data analysis and constructing inferential engine.

The paper presents the main ideas and principles of the Bootstrap technology application for field data processing. Theoretical inferences and clear engineering approach to the estimation of widely used RAM parameters using Bootstrap approach are described in the paper.

Traditional methods of applied statistical analysis are based on the prior hypothetical models (distributions) and suppositions about items under analysis. These assumptions sometimes appear to be not adequate to actual state which drives to untrue results. For reliability analysis it is extremely important due to lack of field data: small samples, non-homogeneity, censoring, etc. Two examples of such errors are shown in Appendix.

The main advantage of the Bootstrap technology is to provide precise enough decision of required problems and to avoid rude mistakes as you do not need to issue non-relevant suppositions.

Upon obtaining arbitrary data sample (from experiment, testing, field, etc.) the Bootstrap allows to generate as many additional samples as necessary, with absolutely the same statistical characteristics which offers the potential for accurate estimation methods and test of hypothesis.

1. Theoretical Concept

The main idea behind the Bootstrap technology is reproduction of the origin sample in order to obtain a required number (it can be very large!) of samples with identical statistical properties. This idea is close to the concept of Maximum Likelihood: data we obtained is the maximum we can rely upon. Such manifold data allows to identify algorithms (rules) for solution of given problems and getting required estimates.

Bootstrap resampling methods are based on the two principles of mathematical statistics, that is: "Convergence of an empirical distribution to the theoretical one", and "Reproduction ability of Monte Carlo Simulation of random variable values for a given distribution".

An observed data sample of size n defines the following empirical distribution, $F_E^{(n)}(t)$:

$$F_B^{(n)}(t) = \sum_{i=1}^{n} w_i e(t-t_i)$$

where t_i is a member of an ordered set of time sample under analysis (TTF, TTR, etc.);
w_i is a corresponding frequency;

$e(t-t_i)$ is a unit step function,

i.e. $e(t-t_i)=0, for \quad t<t_i$ and $e(t-t_i)=1 \quad for \quad t\geq t_i$

The simplest theoretical Monte Carlo procedure of generation of new samples with the identical to the original sample characteristics (Cdf = F_B) is as follows.

First, the n uniformly distributed integers over the segment [1; n] are generated as usually in Monte Carlo simulation. Let us consider them as numbers of elements of the original data sample.

In accordance with the just identified n numbers we create a new so called "Bootstrap sample" from the original one in the following way. Assume, n_1 (where $1 \leq n_1 \leq n$) is the first random number, then the first element of the new Bootstrap sample should be the element number n_1 of the original sample. If n_2 is the second random number, then the second element of this new Bootstrap sample should be the element number n_2 of the original sample, and so on until we get the new sample of n elements. Such generation process (it is named "Bootstrapping") of obtaining samples of size n should be performed the given number of times B.

Basing on the received large number B (10^2 - 10^6) of samples one can perform a test of hypothesis, select a regression equations, determine an optimal replacement policy, derive accurate estimations and confidence intervals for all the necessary Reliability and Maintainability characteristics and parameters (such as Time to Failure distribution, MTBF, MTTR, Probabilities, Up-Time Ratio, Reliability Growth model parameters, etc.).

2. Bootstrap Application Examples for RAM Analysis

To demonstrate the necessity of the Bootstrap technology, let us consider the typical Bootstrap approach to solving such difficult problems as building the confidence intervals for various RAM parameters (statistics), selecting an adequate time to failure/repair distribution model and Reliability Growth (RG) regression function.

2.1 *Building the confidence intervals for various RAM parameters*

Assume, for the given original sample, B Bootstrap samples were generated. For each of these samples we calculate a point estimate $\hat{\theta}$ for a parameter θ under analysis, for example, $\overline{MTBF}=\theta=\sum_{i=1}^{n} t_i/n$.

We comprise an ordered set of received θ statistics for all **B** Bootstrap samples, that is:

$$\theta_1 \le \theta_2 \le \ldots \le \theta_B .$$

As usual, the low $\gamma \cdot 100\%$ confidence limit for MTBF is to be found. With this purpose, we calculate $r_{low}=ent[B(1-\gamma)+1]$, i.e. the integer part of an expression $[B(1-\gamma)+1]$. The integer r_{low} gives a number (index) of an element of the ordered θ set obtained from the generated **B** Bootstrap samples. Obviously, this element $\overline{MTBF}_{r_{low}}=\theta_{ent[B(1-\gamma)+1]}$ is a required result.

Unbiased pivotal $\gamma \cdot 100\%$ confidence interval is defined as:

$$(\overline{MTBF}_{j-ent[\frac{\gamma(B+1)}{2}]} ; \overline{MTBF}_{j+ent[\frac{\gamma(B+1)}{2}]}).$$

where the index j should be defined as a number of element $\overline{MTBF}_j$ which is the closest one to the θ_0, where θ_0 is MTBF of the original sample.

Any required confidence bounds for arbitrary RAM parameters not related with a distribution model (MTTR, Probabilities, Up-Time Ratio, etc.) can be found in the similar way.

2.2 *Selecting an adequate distribution model*

The possibility to define the confidence intervals for summary statistics (such as moments about origin, central moments, cumulants, etc.) allows to solve a problem of selection of the most adequate distribution model from the given distribution set. This problem cannot be resolved by the traditional methods which are only hypothesis tests for goodness of fit.

The solution is based on the well known property of distribution of limited random variable: If two distributions have close values of the first 4 to 6 moments then they are very similar to each other and can approximate each other.

The selection algorithm is as follows.

We define the first 4 to 6 empirical moments for the received **B** Bootstrap samples. For each moment a nested set of central confidence intervals (straps) is calculated. For each type of distributions under analysis corresponding parameters and moments are defined on the base of the original sample.

The most fitting distribution is one whose moments are found in the most narrow confidence intervals. Experimental testing of this method approved that no error occurs even for the small original samples when $n \geq 7$.

2.3 Approach to Reliability Growth analysis

The Bootstrap technology gives an effective solution to a problem of selection of the most fitting regression also. In the RAM analysis they are defined as functions of time. For example, the Reliability Growth analysis requires to estimate changing of MTBF and its VAR over the initial and next time periods. Selection of the most fitting model is performed in accordance with the given criteria.

Theoretical estimates of required statistics can be calculated for each period of time. The differences between empirical point estimates and corresponding results of given regression models define errors for each period of time. The Bootstrap allows to generate as many error sets as necessary. Using these multiple error sets it is possible to identify an optimal regression model in accordance with minimal average or maximal total error, or minimal confidence error area, etc.

Assume, we have made a decision what Reliability Growth model is to be used - Duane or AMSAA. The Bootstrap technology allows to evaluate the accuracy of estimates of model's parameters and the model itself! Let us consider, for instance, the Duane model, i.e. $MTBF_c=T^\alpha/K$. The Bootstrap permits to define the $\gamma \cdot 100\%$ confidence interval for the growth rate α and, hence, the corresponding confidence interval (in the tube form) for the Cumulative MTBF and test time.

The similar method can be applied for evaluating arbitrary regression models, including all models of MIL-HDBK-217 for electronic components reliability.

3. Bootstrap for Censored Samples

Considered Bootstrap methods are applied to complete samples. However, in real life such RAM data as TTF, TTR, and others is not complete and, as a rule, only partially available. In other words, field data samples are censored. The main reasons for this are as follows:

- high inherent reliability,
- limitation in time and expenses,
- changing of test or operation conditions for some items under analysis,
- impossible to continue observation after warranty or contract time,
- need for urgent reliability estimates, and so forth.

Methods of statistical analysis of censored samples differ significantly from classical approaches for complete ones and do not provide approved solution of required problems. The Bootstrap technology permits to solve such problems.

We will demonstrate the Bootstrap application to analysis of censored samples of failure intervals.

Let we have multiple censored sample, including n values t_i of two types:

- true failure intervals (some of them - after repair or removal), i.e. non-censored data,
- elapsed time of products from the beginning of their operating or after repair, i.e. censored data.

We should sort all these values in ascending order. All censored values are marked with a certain score. If some censored values are occasionally equal to the non-censored ones, then they should be placed after the censored values.

The received ordered set defines a part of an empirical Cdf $F_{BC}^{(n)}(t)$:

$$F_{BC}^{(n)}(t)=\sum \frac{\nu_i-\nu_{i-1}}{n+1} e(t-t_i) \quad \text{or} \quad F_{BC}^{(n)}(t)=\frac{\nu_i}{n+1}$$

where $\nu_i=\nu_{pi}+\frac{n+1-\nu_{pi}}{n+1-i}$

i -	is a current number of the ordered set of non-censored value t_i,
ν_i -	is a conditional number of a current non-censored value t_i,
ν_{pi} -	is a conditional number of the previous non-censored value t_i.

The range of random variables is $t_{min} \le t \le t_{max}$ where t_{min} and t_{max} are minimal and maximal non-censored values of the given sample, $\nu_0 = 0$.

The main Bootstrap procedure for the censored samples is similar to the above described one for the complete data. On the base of sampling integers uniformly distributed over the segment [1; n] we identify numbers of values in accordance with the original sample. Using these numbers we build new Bootstrap samples (each of size **n**) where the censored values are marked.

If in some generated sample all values appear to be censored, it is not considered at all. The Bootstrap method allows to generate the given quantity **B** of required samples very easy.

A part of an empirical Cdf matches each of these samples:

$F_{BC(j)}^{(n)}(t),\ j=\overline{1,B}$

Let us consider two methods of definition of frequency and fractile confidence intervals for the given censored samples. We'll define γ confidence intervals for arbitrary non-censored values t_i and its cumulative probability $F_{BC}^{(n)}(t_i)$.

The 1-st Method

We generate Bootstrap samples and consider only those including at least one non-censored value t_i. For each of these values we calculate a corresponding estimate $F_{BC(j)}^{(n)}(t_i)$. The quantity of required Bootstrap samples is defined by the given number **B** of the received values t_i.

Similar to above case, for the given confidence level γ here we should build an ordered set of estimates $F^{(n)}_{BC(j)}(t_i)$, $j=\overline{1,B}$ and calculate the central, pivotal, the shortest or one-side confidence interval.

The γ confidence bounds for the value t_i itself are defined as reciprocal function of the Cdf part $F^{(n)}_{BC}(t)$ for the closest bounds of the frequencies.

The 2-nd Method

We consider such generated Bootstrap samples that include non-censored values t_{kj} fitting the following equality:

$$F^{(n)}_{BC(j)}(t_{kj}) = F^{(n)}_{BC}(t_i)$$

The Bootstrap procedure should be continued until we receive the given number **B** of such samples. We again sort the values $t_{kj}, j=\overline{1,B}$ in ascending order and calculate required confidence intervals for fractiles t_i; corresponding intervals for the probability $F^{(n)}_{BC}(t_i)$ are defined as values of the source empirical function $F^{(n)}_{BC}(t)$ for those argument values equal to received estimates of confidence bounds of t_i.

Similar to the case of complete data, censored samples provide selection of the most fitting distribution from the given distribution set. To do this, the censored empirical CDF should match the original part of Cdf $F^{(n)}_{BC}(t)$, that is:

$$F^{(n)}_{TBC}(t) = \frac{1}{F^{(n)}_{BC}(t_{max}) - F^{(n)}_{BC}(t_{min})}\left[\sum \frac{\nu_i - \nu_i - 1}{n+1} e(t-t_i) - F^{(n)}_{BC}(t_{min})\right]$$

Similarly the censored Cdf $F^{(n)}_{TBC(j)}(t), j=\overline{1,B}$ matches $F^{(n)}_{BC(j)}(t)$ for all Bootstrap samples.

For each of considered Cdf the first 4...6 empirical moments about origin, central moments or some other summary statistics can be easily defined.

On the base of the ordered sets of these statistics a system of nested central confidence intervals is built in accordance with the given sequence of confidence levels $\gamma_1 > \gamma_2 > \ldots$. Then as for the complete data case, for each theoretical Cdf F_T (t) truncated in the points t_{min} and t_{max}, i.e. $F_{TT}(t) = [F_T(t) - F(t_{max})] / [F(t_{max}) - F(t_{min})]$, values like summary statistics should be defined. That theoretical distribution whose moments have the most narrow confidence intervals, should be chosen as the best one.

The received truncated Cdf F_{TT} (t) defines also the complete theoretical Cdf F_T (t), which allows to define any required R M parameters.

4. Conclusion

The Bootstrap is a distribution-free method and it allows to eliminate main disadvantages of the non-parametric statistic, as well as encourages the parametric analysis as a problem of identifying on adequate distribution cannot be solved by the traditional statistic.

Certainly, the Bootstrap cannot discover any statistical property that are absence in the original sample, but it allows to utilize all information containing in the original sample. Moreover, the Bootstrap provides discovering any non-homogeneity of the original sample and testing various hypotheses about this sample very easily and visually.

The new possibilities of the Bootstrap technology allow to upgrade some existing and standardized statistical methods of reliability analysis and develop specific software support - - Bootstrap RAMCAD. Practical experience shows that an appropriate Bootstrap software for the RAM analysis significantly extends Bootstrap applications, increases its accuracy and accelerates the result calculation.

Advanced Logistics Developments (A.L.D. Ltd.) is one of the first developers of the state-of-the-art software package based on the Bootstrap technology.

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APPENDIX

Example of Paradox with Reliability Distribution Models

Goodness-of-Fit Test showed acceptable conformance of received sample data (see Reliability Data Table) to the following distributions with the identical expected value: EXPONENTIAL, WEIBULL, LOG-NORMAL, and FRECHET.

Standard parameters of these distributions were estimated using Probability Paper technique.

The results obtained support the necessity of correct and consistent identification of reliability distribution laws.

Result Table

Distribution and its Parameters	$\theta_{0,999}$, (hours)
EXPONENTIAL (λ = 0.0023)	3003
WEIBULL ($\lambda = 7.5 * 10^{-7}$, b = 2.3)	1066
LOG-NORMAL (m = 6.11, σ = 0.28)	1070
FRECHET ($\lambda = 2.96 * 10^{6}$, b = 2.5)	6142

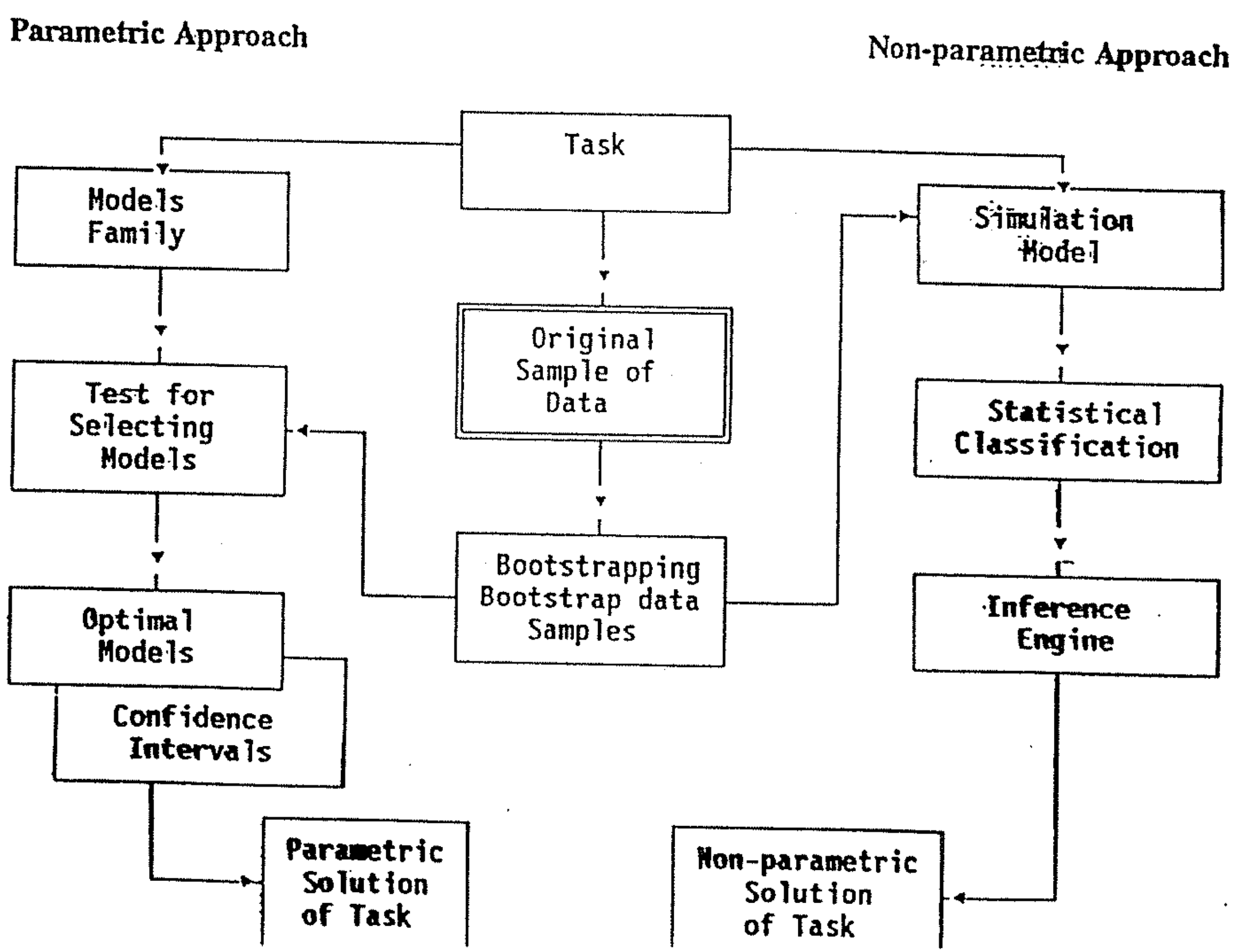
Reliability Data Table

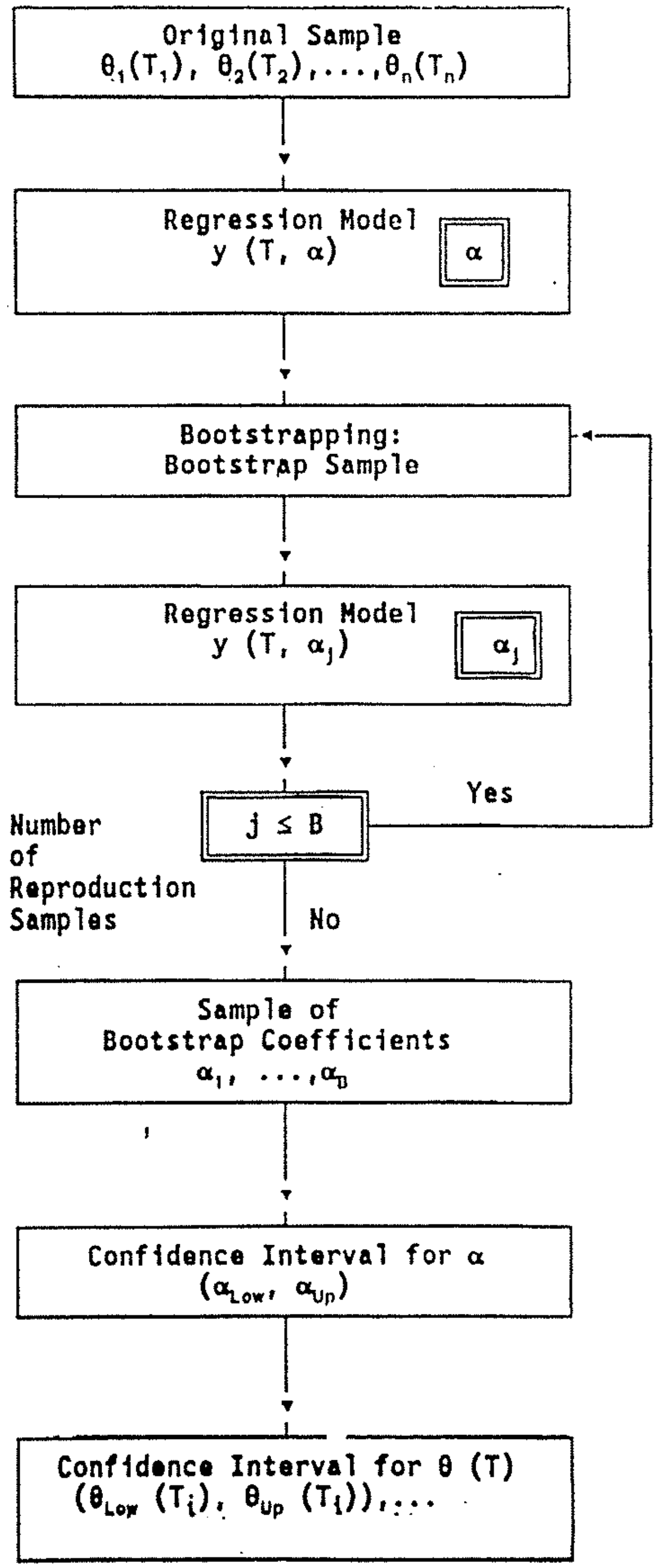
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
t_i,h	160	210	220	240	280	280	285	290	310	370	380	390	390	410
$e^*(t_j)$	0,025	0,050	0,075	0,100	0,125	0,150	0,175	0,200	0,225	0,250	0,275	0,300	0,325	0,35

i	15	16	17	18	19	20	21	22	23	24	25	26	27
t_i,h	440	450	450	460	460	470	480	500	540	545	580	605	610
$e^*(t_j)$	0,375	0,400	0,425	0,450	0,475	0,500	0,525	0,550	0,575	0,600	0,625	0,650	0,675

i	28	29	30	31	32	33	34	35	36	37	38	39	40
t_i,h	680	690	700	710	710	750	75[illegible]	780	880	1050	1100	1400	2000
$e^*(t_j)$	0,700	0,725	0,750	0,[illegible]5	0,800	0,825	0,85[illegible]	0,875	0,900	0,925	0,950	0,975	1,00

Bootstrap Technology
General Approach to RAM Decision Making




Original Sample
θ1(T1), θ2(T2),...,θn(Tn)
Regression Model
y (T, α)
α
Bootstrapping:
Bootstrap Sample
Regression Model
y (T, αj)
αj
j ≤ B
Yes
No
Number
of
Reproduction
Samples
Sample of
Bootstrap Coefficients
α1, ...,αB
Confidence Interval for α
(αLow, αUp)
Confidence Interval for θ (T)
(θLow (Ti), θUp (Ti)),...

Bootstrapping for Models Selection