NEW CHARTS FOR VARIABLES: THEORY AND PRACTICE

Z. Bluvband, ALD Ltd.
P.O Box 679
Rishon Lezion, 75106

P. Grabov, QA&R, Technion
Technion-City
Haifa, 32000

D. Ingman, QA&R, Technion
Technion-City
Haifa, 32000

ABSTRACT

The work examines the conventional charting technique for variables and shows that the Shewhart control charts cannot be considered as optimal. It presents an alternative technique based on constructing an elliptical control region on a two-dimensional graph of the measures of process central tendency and spread. The proposed technique implies setting up a single 'pooled' chart on which the plot of the sample averages is combined with the double variable control limits depending on the sample standard deviations (or the sample ranges). The inner couple of limits is used for process stability evaluation, whereas the outer couple serves for continuous process quality assessment. The chart allows to reduce the process adjustment errors compared with the conventional charts, and shows the process dynamics from both the stability and capability points of view.

INTRODUCTION

Control charts suggested and developed by Shewhart (1931) serve the primary purpose of continuously monitoring whether the manufacturing process is under statistical control, i.e. whether the observed process fluctuations are caused by its inherent variability or by significant changes in the controllable factors (materials, equipment, technology, etc.). The charting technique for measurable controlled characteristics (variables) is tacitly based on the assumption of process normality, hence sample averages ($\bar{x}$) are used to keep track of centering and either the sample ranges (R) or the sample standard deviations (s) are used to watch process variability. All control charts have a common structure. The results of repeated sampling are plotting on a vertical scale against the sample number plotted horizontally. The chart centerline represents the statistic long-term average of the process or its standard value. The upper and lower control limits represent the boundaries of statistic variation. Points falling outside the control limits call for process adjustment.

The work illustrates that Shewhart technique is equivalent to construction of a rectangular control region on a two-dimensional graph formed by the superimposing charts. It is shown in the work that the true control region has an elliptical shape and represents the result of cutting the $\bar{x}$-R or $\bar{x}$-s joint sampling distribution by the horizontal plane at the height corresponding to given significance level. The elliptical shape of the constructed control region dictates rejection of the conception of constant control limits on the charts intended for process control (see Grabov and Bluvband (1994)). The developed charting technique for process stability testing implies setting up the $\bar{x}$-chart with variable control limits depending on the sample standard deviation.

The conventional strategy for process analysis implies a decision concerning not only its stability but also its uniformity, i.e. the ability of the process to yield products with the same properties. Uniformity is evaluated by means of a capability study performed by a periodical auditing of the full set of special collected data. The study involves periodical static estimations in contrast to the continuous procedure of charting, representing therefore a more appropriate technique for uniformity monitoring in the course of the process. The work describes the suggested charting technique for simultaneous analysis of process uniformity. The approach is based on modern capability assessment by means of the Taguchi loss function.

Simultaneous process analysis from both the stability and uniformity points of view can be performed using the proposed single $\bar{x}$-chart with double variable control limits ('pooled' chart) depending on the sample standard deviation. The inner couple of the limits, computed from the equation of the control region boundary, is used for stability evaluation. The outer couple of the limits corresponds to the extreme level of the process loss under random statistical behavior and is used for uniformity evaluation. The chart reduces the risk of adjustment errors versus the Shewhart charts.

CHARTING TECHNIQUE FOR PROCESS STABILITY TESTING

The Shewhart charting technique can be illustrated with the aid of its graphical description presented in Fig. 1, where the $\bar{x}$-s is obtained by the superimposing charts. The control limits plotted on the corresponding axis form a rectangle (referred to as the Shewhart rectangle) representing the control region: for all the points falling within the Rectangle the process is considered to be in a state of statistical control.

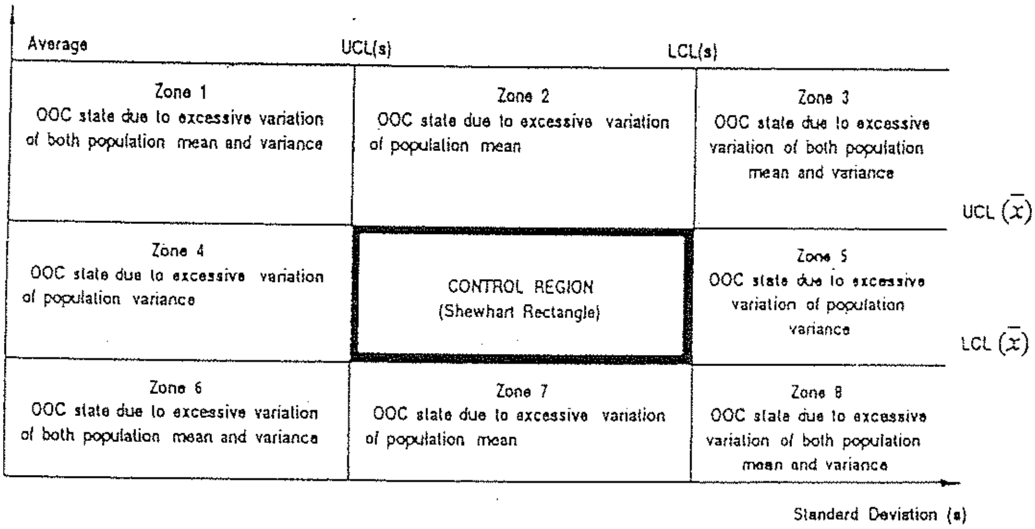

Figure 1. Graphical description of the variables control charts decision making method (OOC denotes "Out-of-Control" state)

The true control region can be obtained by analysis of the joint sampling $\bar{x}$-s distribution, shown in Fig. 2. On the assumption that samples of size n are drawn from a normal universe with parameters μ and σ, the Probability Density Function (PDF) of this distribution can be written as follows (Neyman and Pearson (1928))

$$(1)\quad p(\bar{z}, s^*) = \frac{1}{\Gamma\left(\frac{n-1}{2}\right)} \sqrt{\frac{(n-2)^{n-1}}{\Pi 2^{n-2} \exp(n-2)}} \exp\left\{-\frac{n\bar{z}^2 + (n-2)\left[(s^*)^2 - 2\ln(s^*) - 1\right]}{2}\right\}$$

where $\bar{z} = \frac{\bar{x}-\mu}{\sigma}$ and $s^* = \frac{s}{\sigma}\sqrt{\frac{n-1}{n-2}}$.

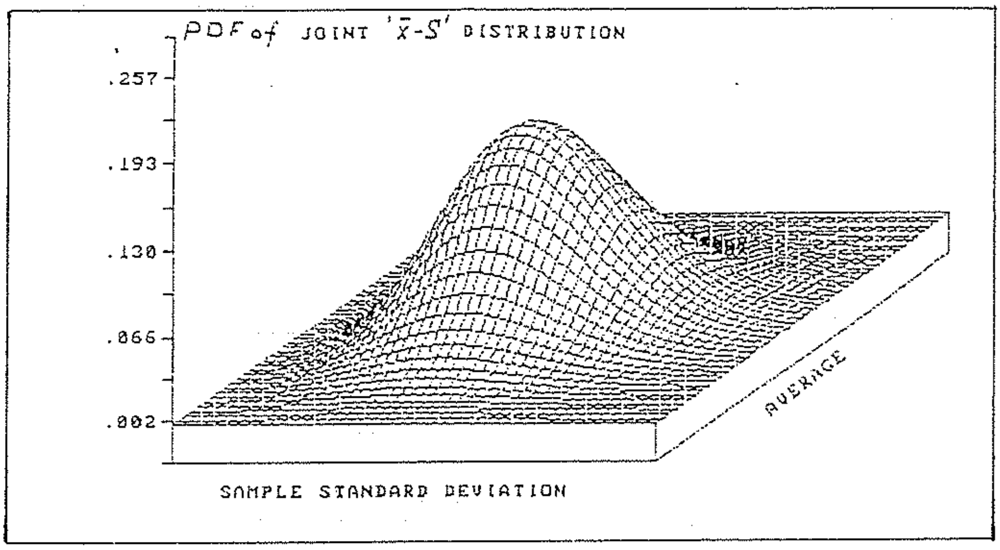

Figure 2. Joint $\bar{x}$-s sampling distribution for n=5

The control region S in the sample space at the significance level α can be defined through the probability $1-\alpha$ of observing a point $Y(\bar{x},s^*)$ with the PDF equal to p(Y) within S, given that H_0 (the General Null Hypothesis: "process is under statistical control") is true, i.e.

(2) $$P(Y\in S|H_0) = \iint p(Y)\, d\bar{z}ds^* = 1-\alpha$$

The control region boundary represents the locus of the sampling points with the same PDF value. The iso-PDF contour bounding the control region corresponds to the same value of the exponent power in braces, which can be designated as the B-statistic

(3) $$B=0.5\,[n\bar{z}^2 + (n-2)\,[(s^*)^2 - 2\ln(s^*) - 1]]$$

It can be shown that the distribution of the B-statistic can be approximated by the exponential distribution: p(B)=exp(-B). The probability of finding a sample point inside the control region given by Eq. (2) can be written as follows

(4) $$\int_{B_{min}}^{B_{crit}} p(B)\, dB = \int_0^{B_{crit}} \exp(-B)\, dB = 1-\alpha$$

where $B_{min}=0$ (the peak of the joint distribution with the coordinates ($\bar{z}=0$, $s^*=1$) and B_{crit} corresponds to the contour bounding the control region. From Eq. (4) it follows that B_{crit}-value depends only on the accepted α: $B_{crit}=-\ln(\alpha)$. Since it is customary to use $\alpha=.0027$ for the conventional control charts, one can get for the joint distribution: $\alpha=1-(1-.0027)^2=.0054$ and $B_{crit}=5.22$.

Obviously, rejection of the rectangular control region inevitably leads to rejection of the charting technique based on constant control limits. The acceptance region for the given $\bar{x}$-value is obtained by cutting the elliptical control region by a perpendicular to the s-axis raised at the point corresponding to the given s-value. Taking in to account that $\bar{x}_i$ and s_i for the boundary contour are related by Eq. (3) and that for pairs $(\bar{x}_i,s_i)$ falling within the control region $B_i \le B_{crit}$ - substitution of the critical B-value on the left-hand side of Eq. (3), yields the equations for the $\bar{x}$-chart limits as function of s_i

(5) $\mu \pm \sigma\sqrt{|K_i|}\, sgn(K_i)$

where $K_i = \dfrac{(n-2)\,[1+2\ln(s_i^*) - (s_i^*)^2] - 2\ln(\alpha)}{n}$

For the so-called 'No Standard Given' case the corresponding estimates ($\bar{s}$ and $\bar{\bar{x}}$) of the population parameters are used in Eq. (5) instead of σ and μ. The above expression implies continuous revision of the control limits in accordance with variation of the s-value, unlike the constant acceptance region on the conventional $\bar{x}$-chart. The variable limits synchronously follow the s-value fluctuations: as the sample standard deviations concentrate in the vicinity of s_{mode}, the limits become wider apart. The closer the s-value to the extreme coordinates of the boundary contour the tighter the $\bar{x}$-chart limits. Overlap of the limits indicates the s-value exceeding these coordinates.

CHARTING TECHNIQUE FOR PROCESS UNIFORMITY TESTING

Stability and uniformity are linked in the sense that they describe the same process, hence uniformity monitoring should be based on the same samples providing information as for stability evaluation. Uniformity can be characterized by a quality loss occurring when the process deviates from its expected (or desired) value and generates non-uniform products. The object of the control charts is to center the process in the vicinity of the chart centerline and to keep its variation. Thus this loss is associated with the second moment about μ ($\bar{\bar{x}}$ for the case 'No Standard Given'), whose value depends on the sample variance and the squared distance from μ to the same average:

(6) $$\frac{1}{n}\sum_{j=1}^{n}(x_{ji}-\mu)^2 = \frac{(n-1)}{n}s_i^* + (\bar{x}_i-\mu)^2 = \sigma^2\left[\frac{n-2}{n}(s_i^*)^2 + \bar{z}_i^2\right]$$

where x_{ji} denotes the j-th reading of the i-th sample. This statistic, named Loss Estimate (LE), can be used for graphic presentation of quality history (see Bluvband and Grabov (1994)). Obviously, the LE-statistic, defined as quality measure of the inherent process variability, closely resembles Taguchi's Expected Loss (EL) characterizing quality of a normally distributed product $N(\mu,\sigma)$ in a symmetrical tolerance (Taguchi et al. (1989))

(7) $$EL = \frac{A}{d^2}\,[\sigma^2 + (\mu - m)^2] \quad ,$$

The coefficient A/d^2 (A denotes the cost of nonconforming unit and d is the half-range of the tolerance) converts the loss into a monetary value and is irrelevant from the monitoring viewpoint. The main difference between Eqs. (6) and (7) is that the EL-value is estimated in relation to the specification target (m) and uses population parameters for loss evaluation, whereas its current counterpart is associated with the loss evaluated in relation to the chart centerline via the sample statistics.

The most critical decision in chart design is specifying the control limits. In our case the limits should characterize the inherent process variability and can be established by comparing of the LE-values corresponding to the extreme points of the control region

- for the highest and lowest points:

$$(8) \quad LE_{h,l} = \sigma^2 \left\{ \frac{n-2[1+\ln(\alpha)]}{n} \right\}$$

- for the rightmost point:

$$(9) \quad LE_r = \sigma^2 \frac{n-2}{n}(s_r^*)^2 = LE_{h,l} + \sigma^2 \frac{n-2}{n} 2\ln(s_r^*)$$

where s_r^* is given by the larger root of the equation: $(s^*)^2 - 2\ln(s^*) = 1 - \frac{2\ln(\alpha)}{n-2}$

Since $s_r^* > 1$, $LE_r \geq LE_{h,l}$ for any sample size (equality corresponds to n=2) and the iso-loss contour on the $\bar{x}$-s graph represents a semiellipse whose center coincides with the origin of coordinates and whose major semiaxis equals s_r^*. Using Eqs. (6) and (9) one can get the control limits on the $\bar{x}$-chart as function of s_i in uniformity testing

$$(10) \quad \mu \pm \sigma\sqrt{|U_i|}\,sgn(U_i)$$

where $U_i = \frac{n-2}{n}\left[(s_r^*)^2 - (S_i^*)^2\right]$.

COMPLETE PROCESS ANALYSIS. 'POOLED' CHART

The control region (B_{crit}= 5.22), the Shewhart Rectangle (n=5) and the iso-loss semiellipse as well as the results of sampling (generated process with normal random variation N(0,1), set of 1,800 samples, n=5) are shown in Fig. 3.

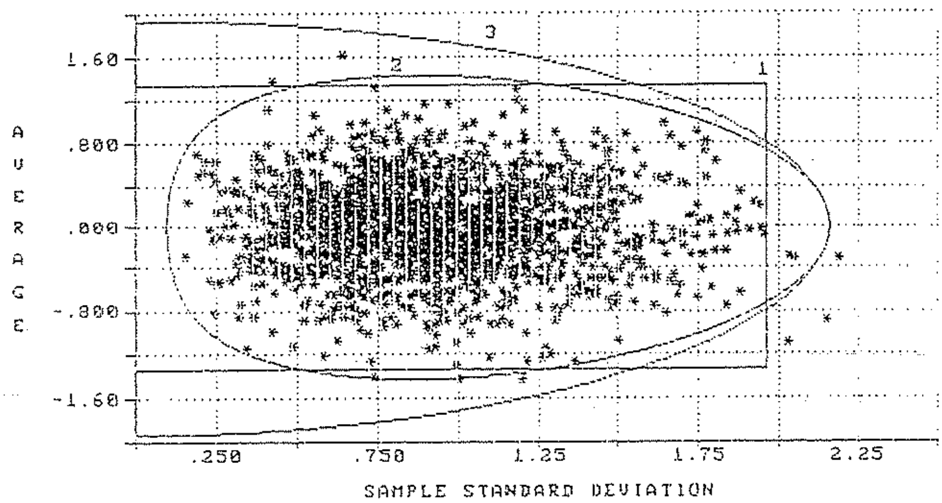

*Figure 3. $\bar{x}$-s graph: * - sampling data; 1 - Shewhart Rectangle; 2 - B-oval; 3 - iso-loss semiellipse*

The area under the $\bar{x}$-s graph is subdivided by the B-oval and the iso-loss semiellipse into three distinct zones, which can be used for complete process analysis (see Fig. 4): within the oval - the process is uniform but unstable, i.e. is uniform today, but may not be tomorrow; outside the simiellipse - the process is both unstable and non-uniform.

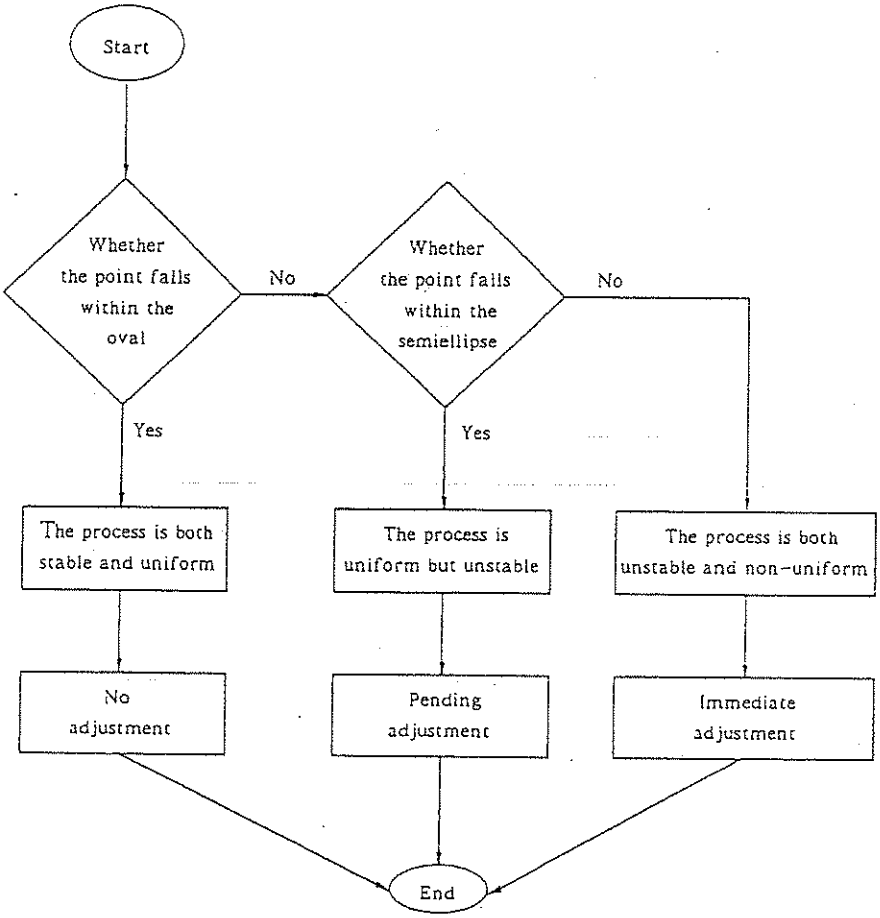

Figure 4. Complete process analysis using the control region and the iso-loss semiellipse

This analysis can be easily performed using a single $\bar{x}$-chart with double variable control limits ('pool' chart): the inner and outer couples of limits are derived in accordance with Eqs. (5) and (10), respectively. This chart contains the information about the behavior of both the central tendency ($\bar{x}$ -plot) and spread (control limits) and allows to perform simultaneous process analysis from both stability (inner limits) and uniformity (outer limits) points of view.

The proposed 'pooled' chart can be standardized by a procedure resembling the technique suggested by Nelson (1989) for control charts with variable sample size. The sample averages are shifted by subtracting the mean (or its estimate) and the result is rescaled by dividing by the distance between the control limit value and the chart centerline. This linear transformation yields a value that shows whether the sample point falls within the control region (or the iso-loss semiellipse). Such a procedure yields two plots on the standardized chart (the inner plot for uniformity assessment and the outer one for stability testing) with a centerline of zero and the constant limits at ±1 due to the two couples of limits on the 'pooled' chart. The points on the standardized chart correspond to the values: $\frac{\bar{x}_i - \bar{\bar{x}}}{\sigma\sqrt{U_i}}$ (uniformity assessment) and $\frac{\bar{x}_i - \bar{\bar{x}}}{\sigma\sqrt{K_i}}$ (stability testing).

The proposed charts are illustrated by the presented example of process analysis using actual data.

CASE STUDY

The viscosity of 25 samples of six was measured. The samples were used to set up the $\bar{x}$-s, 'pooled' and standardized 'pooled' charts shown in Figs. 5a, 5b, 5c and 5d, respectively.

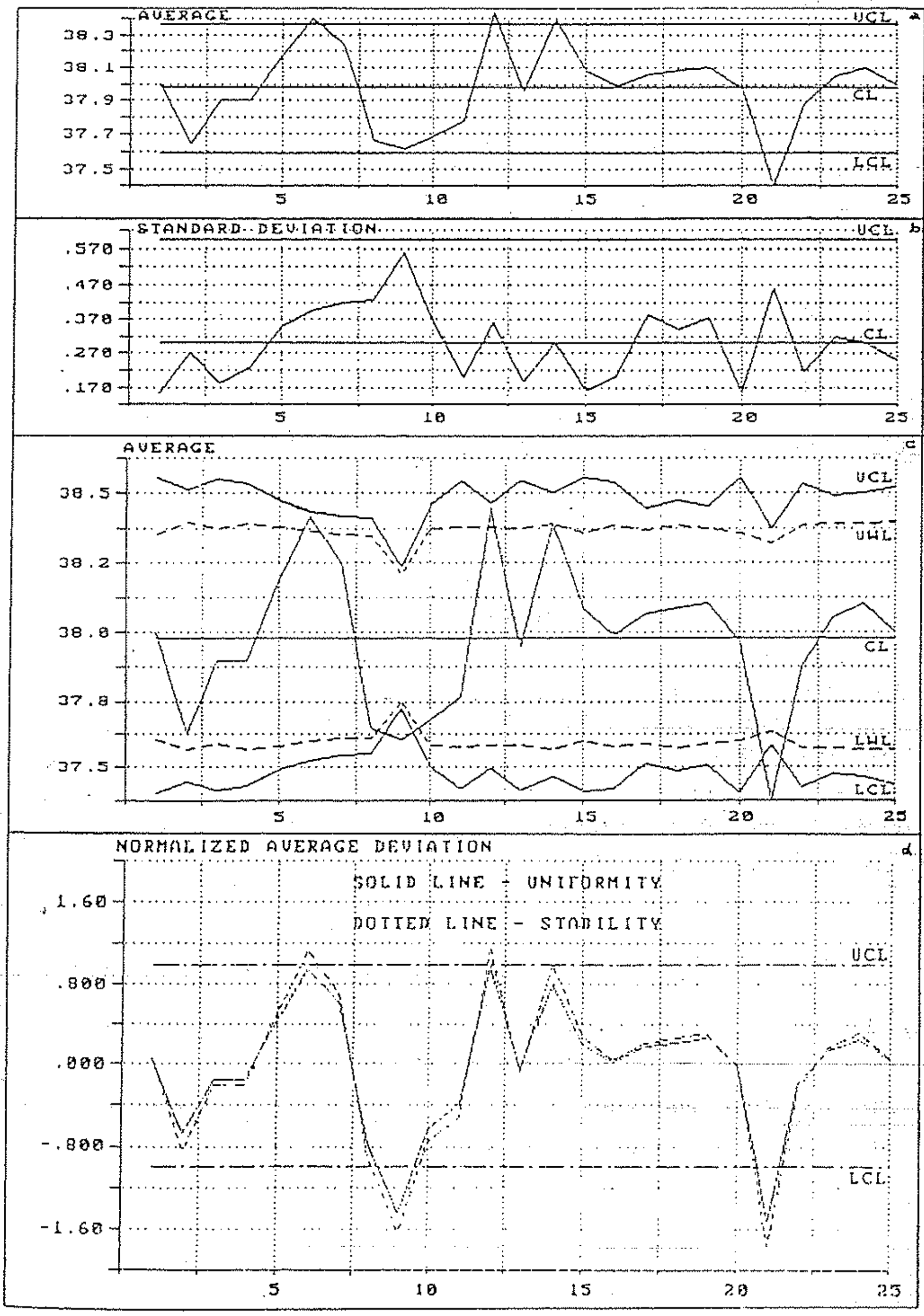

Figure 5. Case study charts: averages - a, standard deviations - b, 'pooled' - c, standardized 'pooled' - d (UWL and LWL on the 'pooled' chart denote upper and lower warning limits, respectively, used for process stability testing)

Comparative analysis of the conventional and proposed charts shows that process management according to the former leads both to overadjustment (point 14 - from both the stability and uniformity points of view; points 6 and 12 - from the quality point of view only), and to underadjustment (point 9), when no appreciable change due to assignable cause (simultaneous shift of the process mean and increase of the process variability) had been detected.

CONCLUSION

Practitioners sometimes criticize feedback controllers operating in conjunction with the Shewhart charts for overcompensating disturbances. The probability of overadjustment as well as the risk of underadjustment can be reduced by using the suggested approach allowing to apply the charting technique for simultaneous analysis of both statistical and quality process behavior and, thereby, optimize process control. The additional advantage of the suggested chart is that it does not require a constant sample size for setting up.

The suggested technique is characterized by some charting complexity compared with the Shewhart charts. However, modern quality-control equipment directly transfers the observation data to a computer where a suitable program performs the statistical calculations and displays the control chart on a monitor. A program corresponding to the developed new charting technique is available.

BIBLIOGRAPHY

Bluvband, Z. and Grabov, P. (1994). 'Process Control by Quality Charting'. Proceedings of the 3-rd IEEE Conference on Control Applications, the University of Strathclyde, Glasgow, Scotland, U.K, August 24-26, pp. 1259-1263.

Grabov, P. and Bluvband, Z. (1994). 'Statistical Process Control: New Charting Technique'. Proceedings of the International Conference 'Statistics in Industry, Science and Technology', the University of Tokyo, Japan, July 11-13, pp.161-166.

Nelson, L. S. (1989). 'Standardization of Shewhart Control charts'. Journal of Quality Technology, 4, pp. 287-289.

Neyman, J. and Pearson, E. S. (1928). 'On the Use and Interpretation of Certain Test Criteria for Purposes of Statistical Interference'. Biometrica 20A: 175-240.

Shewhart, W. A. (1931). 'Economic Control of Quality of Manufactured Product'. Reprinted by American Society for Quality Control, 1981, Milwaukee, WI.

Taguchi, G., Elsayed, E. A. and Hsiang, T. (1989). 'Quality Engineering in Production Systems', McGraw-Hill International Editions.